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A THREE-DIMENSIONAL SELF-SIMILAR PROBLEM OF SUPERSONIC WEDGING OF AN ELASTIC BODY*

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A method based on the dynamic theory of elasticity is proposed for solving the three-dimensional problem of supersonic wedging a body by a thin arrow-shaped blade. Using appropriate scale transformation and passing to limit ("the microscope principle" /1/), the problem of the thin blade is reduced to that of the mathematical slit. A solution in closed form is obtained in the case of dependence of displacements, or of some differential transform of these, on two self-similar variables. The input problem is reduced by a change of coordinates to the determination of analytic functions of two complex variables which, after some transformations, reduces to the Dirichlet boundary value problem in a single complex variable, which is solvable by conventional methods.

Problems of thin supersonic blades in an elastic body /2/ are of interest in investigations of supersonic cutting and in the theory of electron and laser fracturing of solids. Problems of the supersonic punch moving on a surface also belong to problems of supersonic fracturing. Theoretically all these problems have a general solution which is, however, very difficult to obtain with the use of integral representations /3/.

1. Statement of the problem.

Let an infinite thin blade — mathematical slit in the plane x_1, x_3 — symmetric about the x_3-axis move in an elastic body along the x_3-axis at constant supersonic velocity $V > c_1 > c_2$ (c_1 and c_2 are velocities of longitudinal and transverse elastic waves, respectively). Angle β of the arrow-shaped blade tip (or the coefficient $\gamma = \operatorname{ctg} \beta$) is constant and contained within the cone of characteristics (Fig.1), i.e. the condition

$$\gamma > M_1 > M_2 \quad (M_j^2 = V^2/c_j^2 - 1 > 0 \ (j = 1, 2))$$

is satisfied. The problem is assumed symmetric relative to the plane x_1x_3.

Equations of the dynamic theory of elasticity in displacements /3/ are used here as governing equations. In the steady-state problem they are in the moving system of coordinates $x = x_1,\ y = x_2,\ z = x_3 - Vt$ of the form

$$\frac{\partial^2 \mathbf{u}_j}{\partial x^2} + \frac{\partial^2 \mathbf{u}_j}{\partial y^2} = M_j^2 \frac{\partial^2 \mathbf{u}_j}{\partial z^2} \quad (j = 1, 2) \tag{1.1}$$

$$\operatorname{rot} \mathbf{u}_1 = 0, \ \operatorname{div} \mathbf{u}_2 = 0 \tag{1.2}$$

where $\mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2$ is the total displacement vector, and vectors $\mathbf{u}_1$ and $\mathbf{u}_2$ define, respectively, the dilatation and shear wave displacements.

According to Hooke's law

$$\sigma_{kl} = 2\mu\varepsilon_{kl} + \lambda\varepsilon\delta_{kl};\ 2\varepsilon_{kl} = u_{k,l} + u_{l,k},\ \varepsilon = u_{k,k} \tag{1.3}$$

where σ_{kl} $(k, l = x, y, z)$ are components of the stress tensor, and λ and μ are Lamé constants.

The basic equations (1.1) — (1.3) remain valid after the substitution for $\mathbf{u}$ of function $\mathbf{v} = \mathbf{L}\mathbf{u}$ which represent linear transforms of the displacement field ($\mathbf{L}$ is some linear differential operator), and of the respective substitutions

$$\varepsilon_{kl} \to \mathbf{L}\varepsilon_{lk},\ \sigma_{lk} \to \mathbf{L}\sigma_{lk}.$$

Let us consider the self-similar three-dimensional problems of supersonic arrow-shaped blades (a particular case of the general problem of the supersonic cone) whose respective solutions (for instance displacements) depend only on the two self-similar variables

$$\xi = x/z,\ \eta = y/z$$

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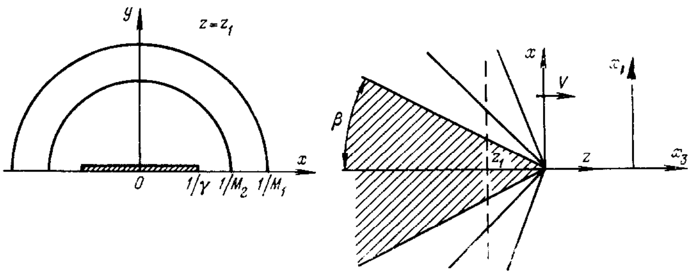

Fig.1

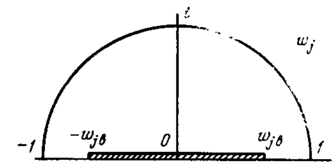

Fig.2

Problems in which displacements u are homogeneous functions of x, y, z of zero measure /4/ and, also, problems in which functions $v = Lu$ (for instance the velocity vector of medium) are homogeneous, belong to this type. The approach developed here for three-dimensional problems is similar to methods of the theory of functions of complex variables in plane static /5/, steady-state dynamic /6/, and dynamic /4/ problems of the theory of elasticity.

Let us solve the problems formulated below.

Problem A. The displacements are homogeneous functions of ξ and η. The boundary conditions in the system of coordinates xyz are of the form

$$u_x = g_x(x/z),\ u_y = g_y(x/z),\ u_z = g_z(x/z) \quad \text{for} \quad |x| \leqslant -z/\gamma,\ y = 0,\ z < 0 \tag{1.4}$$

$$u_{jx} = u_{jy} = u_{jz} = 0\ (j = 1, 2) \quad \text{for} \quad x^2 + y^2 > z^2/M_j^2,\ z < 0 \quad \text{and all} \quad z > 0 \tag{1.5}$$

$$u_x = u_y = u_z = 0 \quad \text{for} \quad |x| \geqslant -z/\gamma,\ y = 0,\ z < 0. \tag{1.6}$$

where u_x, u_y, u_z are Cartesian coordinates of the displacement vector u, g_x, g_y and g_z are given functions of argument $\xi = x/z$, and conditions (1.5) indicate the boundaries of perturbed regions and limit solutions to displacements free of discontinuities at these boundaries. Conditions (1.6) have the meaning of conditions of symmetry which ensure the homogeneity of solutions, and are similar to the usual but weaker conditions of equilibrium (absence of stress discontinuity at the slit). Such conditions obtain, for instance, when a piecewise-homogeneous body is split along the interface of the elastic and absolutely rigid part of a body, or in the case of thin blades with arrow-shaped tips of large apex angle (close to that of the cone of characteristics).

Problem B. Functions $v = Lu$ (linear transform) are homogeneous functions of ξ and η. Since the boundary conditions coincide with (1.4)—(1.6) of Problem A, except for the substitution $u \to v$, hence the two problems differ only in their mechanical aspects, i.e. in the determination of stresses in the medium, while from the point of view of mathematics, as problems of finding the unknown function (u or v) from its value at boundaries, both are exactly the same.

The velocity field of the medium provides a simple example of function v in Problem B; the transform operator is $L = \partial/\partial t = -V\partial/\partial z$.

Condition (1.2) has in coordinates ξ, η the form

$$u_{1z,\xi} + \xi u_{1x,\xi} + \eta u_{1x,\eta} = 0 \tag{1.7}$$

$$u_{1z,\eta} + \xi u_{1y,\xi} + \eta u_{1y,\eta} = 0,\ u_{1y,\xi} - u_{1x,\eta} = 0$$

$$u_{2x,\xi} + u_{2y,\eta} - \xi u_{2z,\xi} - \eta u_{2z,\eta} = 0 \tag{1.8}$$

2. The method of solution : reduction to the Dirichlet boundary value problem. Further exposition is carried out in terms of Problem A (in displacements). The obtained results with the substitution $u \to v = Lu$, $\sigma_{ik} \to L\sigma_{ik}$ are completely valid for Problem B.

We introduce for each of Eqs.(1.1) its proper system of coordinates $r_j\sigma_j\varepsilon_j$ defined by the relations

$$M_j\xi = \cos\sigma_j \frac{2\varepsilon_j}{1+\varepsilon_j^2} \tag{2.1}$$

$$M_j\eta = \sin\sigma_j \frac{2\varepsilon_j}{1+\varepsilon_j^2}; \quad z = r_j \frac{1+\varepsilon_j^2}{1-\varepsilon_j^2}$$

Equations (1.1) with allowance for self-similarity, i.e. independence of solutions from r_j, in coordinates $\sigma_j\varepsilon_j$ assumes the form of the Laplace equation

$$\frac{\partial^2 \mathbf{u}_j}{\partial \sigma_j^2} + \varepsilon_j \frac{\partial}{\partial \varepsilon_j}\left(\varepsilon_j \frac{\partial \mathbf{u}_j}{\partial \varepsilon_j}\right) = 0 \tag{2.2}$$

The method of transforming equations of elastic medium dynamics used here is similar to that applied by Busemann and others in problems of supersonic gasdynamics (see, e.g., /7/).

Vectors $\mathbf{u}_j$ may, therefore, be considered as the real part of some analytic vector function with the corresponding complex variable

$$w_j = \varepsilon_j \exp(i\sigma_j) \tag{2.3}$$

Using the transform with an appropriate selection of the branch in formulas (2.1), the initial perturbations region u_j in space xyz is mapped in the complex plane w_j onto the unit circle interior, and the blade onto the segment with ends at points $w_{jh} = \pm M_j^{-1}(\gamma - \sqrt{\gamma^2 - M_j^2})$, as shown in Fig.2.

We introduce the notation

$$u_{jk} = \operatorname{Re} F_{jk}(\omega_j) \quad (j = 1, 2;\ k = x, y, z) \tag{2.4}$$

where F_{jx}, F_{jy}, F_{jz} are functions of respective complex variables w_1 and w_2 analytic in the unit circle interior with a slit along segment $\operatorname{Im} w_j = 0$, $|\operatorname{Re} w_j| \leqslant w_{jh}$. Substituting expressions (2.4) (for u_{jk}) into (1.7) and (1.8) and taking into account the relations

$$\frac{(w_j)_\xi'}{(w_j)_\eta'} = i\frac{w_j^2+1}{w_j^2-1}, \quad \eta + \xi\frac{(w_j)_\xi'}{(w_j)_\eta'} = \frac{2iw_j}{M_j(w_j^2-1)}$$

we obtain three conditions for the connection between six analytic functions $F_{jk}(w_j)$ $(j = 1, 2;$ $k = x, y, z)$ of the form

$$\begin{aligned} &i(w_1^2+1)F_{1y}'(w_1) = (w_1^2-1)F_{1x}'(w_1) \\ &2iw_1F_{1y}'(w_1) + M_1(w_1^2-1)F_{1z}'(w_1) = 0 \\ &iM_2(w_2^2+1)F_{2x}'(w_2) + M_2(w_2^2-1)F_{2y}'(w_2) = 2iw_2F_{2z}'(w_2) \end{aligned} \tag{2.5}$$

where the prime denotes differentiation with respect to the argument.

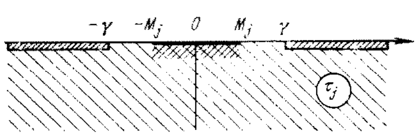

Fig.3

Problem A is thus reduced to a boundary value problem of the theory of analytic functions of two complex variables that can be substantially simplified whenever it is possible to determine the relation between these variables.

Note that only two of the three conditions (1.7) are independent, hence only three independent conditions obtained from (1.7) and (1.8) appear in (2.5). Conditions (2.5) imply that only one function F_{1k} and two functions F_{2k} are independent.

We introduce one more pair of complex variables τ_j defined by the following expression (the Joukowski transform):

$$\tau_j = \frac{M_j}{2}\left(w_j + \frac{1}{w_j}\right) \quad (j = 1, 2) \tag{2.6}$$

In both planes of τ_j the unit circle interior which corresponds to the upper half-plane w_j (the perturbation region) passes into the lower half-plane $\operatorname{Im}\tau_j < 0$, and the blade surface is transformed into two equal half-lines on the real axis $|\operatorname{Re}\tau_j| \geqslant \gamma$, $\operatorname{Im}\tau_j = 0$. Moreover, the circle of unit radius becomes in the w_j plane the segment $|\operatorname{Re}\tau_j| > M_j$, $\operatorname{Im}\tau_j = 0$, and the real axis $\operatorname{Im} w_j = 0$ is represented by two half-lines $|\operatorname{Re}\tau_j| > M_j$, $\operatorname{Im}\tau_j = 0$ (see Fig.3). Consequently, all boundaries at which conditions (1.4) — (1.6) of Problem A are specified in the τ_j plane are represented by the real axis $\operatorname{Im}\tau_j = 0$, while at $y = 0$ $(\eta = 0)$ we have the equality

$$\tau_1 = \tau_2 = 1/\xi \quad \text{for} \quad |\operatorname{Re}\tau_j| \geqslant \gamma,\ \operatorname{Im}\tau_j = 0 \tag{2.7}$$

Components of the total displacement vector of the medium $\mathbf{u}$ assume, in terms of the complex variables τ_j the form

$$u_k = \mathrm{Re}\,[F_{1k}(\tau_1) + F_{2k}(\tau_2)] \quad (k = x, y, z) \tag{2.8}$$

where $F_{jk}(\tau_j)$ are functions which by virtue of symmetry are analytic in the lower half-plane of τ_j and are to be determined. They are linked by supplementary condition which in conformity with formulas (2.5) and (2.6) are of the form

$$\sqrt{\tau_1^2 - M_1^2}\,F_{1x}' = -i\tau_1 F_{1y}', \quad \sqrt{\tau_1^2 - M_1^2}\,F_{1z}' = iF_{1y}'$$
$$\sqrt{\tau_2^2 - M_2^2}\,F_{2y}' = i\tau_2 F_{2x}' - iF_{2z}' \tag{2.9}$$

Components of the strain tensor (formulas (1.3)) are obtained using the following formulas:

$$2\varepsilon_{lk} = \mathrm{Re}\left[\frac{\partial\tau_1}{\partial x_k}F_{1k}' + \frac{\partial\tau_2}{\partial x_k}F_{2k}' + \frac{\partial\tau_1}{\partial x_l}F_{1l}' + \frac{\partial\tau_2}{\partial x_l}F_{2l}'\right] \tag{2.10}$$
$$\varepsilon = \mathrm{Re}\left[\frac{\partial\tau_1}{\partial x}F_{1x}' + \frac{\partial\tau_2}{\partial x}F_{2x}' + \frac{\partial\tau_1}{\partial y}F_{1y}' + \frac{\partial\tau_2}{\partial y}F_{2y}' + \frac{\partial\tau_1}{\partial z}F_{1z}' + \frac{\partial\tau_2}{\partial z}F_{2z}'\right]$$

In formulas (2.9) and (2.10) the arguments of functions F_{jk} are the respective variables τ_j, and the prime denotes differentiation with respect to the argument.

Note that for determining the relation between variables τ_1 and τ_2 when $|\mathrm{Re}\tau_j| < \gamma$, $\mathrm{Im}\,\tau_j = 0$ the boundary conditions (1.5) and (1.6) assume the form

$$\mathrm{Re}F_{1x}(\tau_1) = \mathrm{Re}F_{1y}(\tau_1) = \mathrm{Re}F_{1z}(\tau_1) = 0$$
$$\mathrm{Re}F_{2x}(\tau_2) = \mathrm{Re}F_{2y}(\tau_2) = \mathrm{Re}F_{2z}(\tau_2) = 0 \tag{2.11}$$

i.e. they are zero for any τ_1 and τ_2 along that segment. This can be expressed as follows. For any pair of τ_1 and τ_2 belonging to the respective segments $|\mathrm{Re}\tau_j| < \gamma$, $\mathrm{Im}\tau_j = 0$ the equality

$$\mathrm{Re}\,[F_{1k}(\tau_1) + F_{2k}(\tau_2)] = \mathrm{Re}\,[F_{1k}(\tau) + F_{2k}(\tau)] = 0$$

is always satisfied.

Since this equality is satisfied also for $\tau = \tau_1 = \tau_2$, it is possible to extend condition (2.7) to the indicated segments of real axes of the τ_j planes without altering boundary conditions (2.11). It is, thus, possible to assume that the condition

$$\tau = \tau_1 = \tau_2 \tag{2.12}$$

of equality of the two complex variables is satisfied at all points of real axes $\mathrm{Im}\tau_1 = 0$ and $\mathrm{Im}\,\tau_2 = 0$.

We introduce a new complex variable whose real part is the same as that of variables τ_1 and τ_2

$$\mathrm{Re}\tau = \mathrm{Re}\tau_1 = \mathrm{Re}\tau_2 \tag{2.13}$$

i.e. we extend condition (2.12) of relation between the two complex variables on the real axis to the entire lower half-plane.

The indicated here method of reducing the determination of six functions of two complex variables to a boundary value problem for a function of a single complex variable, is in many respects similar to that used in /4/ for solving several large classes of plane self-similar problems of the dynamic theory of elasticity of subsonic velocities investigated by the authors using the Smirnov—Sobolev method of functionally invariant solutions of wave equations /4,8/.

We introduce the notation

$$F_{1x}(\tau) + F_{2x}(\tau) = V_x(\tau), \quad F_{1y}(\tau) + F_{2y}(\tau) = V_y(\tau)$$
$$F_{1z}(\tau) + F_{2z}(\tau) = V_z(\tau) \tag{2.14}$$

where $V_k(\tau)$ $(k = x, y, z)$ are functions analytic in the lower half-plane $\mathrm{Im}\tau < 0$. In conformity with formulas (2.9) functions $F_{jk}(\tau)$ are expressed in terms of $V_k(\tau)$ as follows:

$$F_{1x}' = -i\tau\varkappa_1 D\,[V_y' + i\varkappa_2 V_z' - i\tau\varkappa_2 V_x']$$
$$F_{1y}' = D\,[V_y' + i\varkappa_2 V_z' - i\tau\varkappa_2 V_x'] \tag{2.15}$$
$$F_{1z}' = i\varkappa_1 D\,[V_y' + i\varkappa_2 V_z' - i\tau\varkappa_2 V_x']$$
$$F_{2x}' = D[i\tau\varkappa_1 V_y' - \tau\varkappa_1\varkappa_2 V_z' + (1 - \varkappa_1\varkappa_2)V_x']$$

$$F_{2y}' = i\varkappa_2 D\,[\tau V_x' - V_z' + i\varkappa_1 (1 + \tau^2) V_y']$$
$$F_{2z}' = D\,[(1 - \tau^2\varkappa_1\varkappa_2) V_z' - i\varkappa_1 V_y' - \tau\varkappa_1\varkappa_2 V_x']$$
$$\varkappa_j(\tau) = (\tau^2 - M_j^2)^{-1/2}\ (j = 1,\ 2),\ D(\tau) = [1 - \varkappa_1\varkappa_2 (1 + \tau^2)]^{-1}$$

where τ is the argument of functions $V_x, V_y, V_z, \varkappa_1, \varkappa_2, D, F_{jk}$.

For the determination of function $V_k(\tau)$ we have three separable independent Dirichlet problems for a single complex variable in the half-plane $\operatorname{Im}\tau < 0$, which in accordance with formulas (1.4)—(1.6), (2.7)—(2.9), and (2.11)—(2.14) are of the form

$$\operatorname{Re} V_k(\tau) = g_k(\xi = 1/\tau),\ |\operatorname{Re}\tau| \geqslant \gamma \qquad (2.16)$$
$$\operatorname{Re} V_k(\tau) = 0, |\operatorname{Re}\tau| < \gamma\ (k = x,\ y,\ z)$$

where g_k which in conformity with (1.4) are known functions of variable ξ ("the blade profile").

The solution of each of the boundary value problems (2.16) is a Schwarz integral for the half plane which, for example, for $V_x(\tau)$ is of the form /9/

$$V_x(\tau) = \frac{1}{\pi i}\int_{-\infty}^{+\infty} \operatorname{Re} V_x(t)\frac{dt}{t - \tau} + iC_x \qquad (2.17)$$

where C_x is an arbitrary real constant and $\operatorname{Re} V_x(t)$ are defined by condition (2.16).

Expressions of form (2.17) for each of the sought functions completely solve Problem A. Displacements are determined by formulas (2.8) and integration of relations (2.15), while the stress field is obtained directly using formulas (1.3) and (2.10).

Problem B is similarly solved using formulas (2.17), except that the field of displacement $\mathbf{u}$ is obtained from the solution of the boundary value problem for field $\mathbf{v}$ using the inverse transform $\mathbf{u} = \mathbf{L}^{-1}\mathbf{v}$ and the stress field is then determined by formulas (1.3), or the linear transform $\mathbf{L}\sigma_{lk}$ is directly obtained from formulas (2.10) and, then, the field of σ_{lk} is determined using transform $\mathbf{L}^{-1}$.

3. One boundary value problem and its applications. We shall illustrate the general method on the example of an important limit case of the boundary value problem (1.4)—(1.6).

Let

$$g_k(\xi) = a_k\ (k = x,\ y,\ z)$$

where a_x, a_y, a_z are some arbitrary real constants. In accordance with (2.17) we have

$$V_k(\tau) = -i\,(a_k/\pi)\ln[(\gamma - \tau)/(\gamma + \tau)] + iC_k\ (k = x,\ y,\ z)$$

where C_x, C_y, C_z are arbitrary real constants. For determining stresses we use formulas (2.10). When $\eta = 0$ we have for functions $F_{jk}(\tau)$ in terms of variables ξ, η in accordance with definitions (2.15) the following expressions:

$$F_{1x}' = -2i\xi H\,(a_x - a_z\xi + ia_y h_2) \qquad (3.1)$$
$$F_{1y}' = iF_{1x}'h_1,\ F_{1z}' = -\xi F_{1x}'$$
$$F_{2x}' = 2i\xi H\,[(h_1h_2 - \xi^2)a_1 - a_3\xi + ia_2h_2] \qquad (3.2)$$
$$F_{2y}' = -2\xi H\,[a_1h_1 - h_1h_2a_3 + ia_2\xi^{-1}h_2(1 + \xi^2)]$$
$$F_{2z}' = -2i\xi H[a_1\xi - a_3(h_1h_2 - 1) + ia_2\xi h_2];\ |\xi| < M_2^{-1}$$
$$F_{2x}' = F_{2y}' = F_{2z}' = 0,\ M_2^{-1} < |\xi| < M_1^{-1} \qquad (3.3)$$
$$h_j = \sqrt{1 - M_j^2\xi^2},\ H = (\gamma^2\xi^2 - 1)^{-1}(h_1h_2 - 1 - \xi^2)^{-1}\quad (j = 1,\ 2)$$

and for the derivatives of complex variables τ_j in terms of coordinates x, y, z and η 0 we have

$$\frac{\partial\tau_j}{\partial x} = -\frac{1}{z\xi^2},\quad \frac{\partial\tau_j}{\partial z} = \frac{1}{z\xi},\quad \frac{\partial\tau_j}{\partial y} = -i\frac{h_j}{z\xi^2}\quad (j = 1,2) \qquad (3.4)$$

The set of Eqs. (3.1)—(3.4) enables us to determine the stress field using formulas (3.1)—(3.4). Omitting the formulas for all of the nine components σ_{lk}, which are readily obtained from (3.1)—(3.4), we present expressions for the first invariant of the stress tensor for $(\eta = 0)$

$$I_1 = 2(2\mu + 3\lambda)a_2 z^{-1}\xi^{-2}h_2H\,[\xi h_1^2 - h_2(1 + \xi^2)],\ |\xi| < M_2^{-1} \qquad (3.5)$$

$$I_1 = \frac{2(2\mu + 3\lambda)(1 + M_1^2)(a_1 - a_3\xi - a_2h_2)h_1h_2}{z\xi(\gamma^2\xi^2 - 1)[2 + M_1^2 + M_2^2 + (1 - M_1^2M_2^2)\xi^2]},\quad M_2^{-1} < |\xi| < M_1^{-1} \qquad (3.6)$$

Let us point out certain singularities of the obtained results.

The stress field in region $\xi^2+\eta^2<M_2^{-2}$ of existence of transverse and longitudinal waves is determined only by the normal displacement a_y at the slit, i.e. the problem of arbitrary displacement discontinuity at the supersonic slit can be considered, without loss of generality, in this region as a problem of normal discontinuity.

In the neighborhood of points $|\xi|=1/\gamma, |\xi|=1/M_2$ and $|\xi|=1/M_1$ the stress field has singularities. The asymptotics of the stress field are indicated below. For the invariant I_1 we have

$$I_1=\frac{(2\mu+3\lambda)a_2\sqrt{\gamma^2-M_2^2}\,[\gamma^2-M_1^2+(1+\gamma^2)\sqrt{\gamma^2-M_2^2}]}{\rho z\,[\sqrt{(\gamma^2-M_1^2)(\gamma^2-M_2^2)}-\gamma^2-1]} \tag{3.7}$$

$$\rho=1-\xi\gamma\to\pm 0$$

As was to be expected, the singularity of stresses (3.7) is of order $1/\rho$, i.e. the same as that near the edge of the subsonic dislocation discontinuity /1,4/. Near the boundaries of perturbation regions (Mach fronts) we have

$$I_1=a_2R_2(M_1^2-M_2^2)\sqrt{\rho},\quad \rho=1-M_2\xi\to+0 \tag{3.8}$$

$$I_1=M_2R_2(1+M_1^2)\sqrt{M_2^2-M_1^2}\,(a_1-a_3/M_2-a_2\sqrt{2\rho})\sqrt{\rho}\quad \rho=M_2\xi-1\to+0$$

$$I_1=R_1\sqrt{M_2^2-M_1^2}\,(a_1M_1-a_3-a_2\sqrt{M_2^2-M_1^2})\sqrt{\rho},\quad \rho=1-M_1\xi\to+0$$

$$R_j=\frac{2\sqrt{2}\,(2\mu+3\lambda)M_j^3}{z(\gamma^2-M_j^2)(1+M_j^2)}\quad (j=1,2)$$

All results of Sect.3 equally apply to problem B with the substitution $\sigma_{lk}\to L\sigma_{lk}$ taken into account. In the important case of operator $L=-V\partial/\partial z$ (the problem of uniform velocities of the medium), similar to the plane problem considered in /2/, we find that near the blade edge the stresses obtained by applying the inverse operator L^{-1} to expressions in (3.7) and (3.8) have singularities of order $\ln(z+\gamma|x|)$.

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